Making money from mathematical models

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This paper is a discussion of the scientific significance and nature of mathematical models generally, and in finance theory in particular; of the relation of such models to a postulated 'reality'; of the sufficiency of the empirical grounding of such models to the task of drawing useful inferences about such a reality; and of the danger of unwittingly propagating erroneous conclusions about this reality in the absence of such a grounding. Some suggestion as to how the current empirical grounding of mathematical modelling in finance can be enriched.

1. Introduction

There is a little remarked upon dissonance between the philosophy of mathematics and that of the natural sciences. Mathematics is an abstract philosophy with an essentially aesthetic nature. Many mathematicians and philosophers have asked whether it is best to describe mathematics as having been discovered or invented. Perhaps more pragmatic mathematicians would be inclined to follow Wittgenstein's advice and pass over in silence that of which they cannot speak! The choice between these options is one with no profound consequences for the practice of mathematics.

Natural science has no such incipient philosophical argument at its heart. The philosophy of natural science may not be generally well understood but it is, to my knowledge, relatively uncontentious.

The natural scientist observes naturally occurring phenomena and attempts first to classify and subsequently to model them (in a theory) by a process of metaphor and analogy. If such a description seems at first puzzling, we must reflect on the logic of the fact that the theory cannot be the reality; it can at best only be a near perfect simulation of that reality. At any time the reality to be modelled consists of the universe of recorded observations, a universe which as long as mankind continues to progress expands so that even a model in accordance with every known observation could not properly be described as 'true' in the sense that it is necessarily identical to the postulated reality. The discipline which, in my opinion, has made the philosophy of natural science so productive for mankind is the agreement that such a model must be capable of producing empirically falsifiable predictions and must thus be capable of being subjected to an unbiased test of its value within its own conceptual framework. A theory's utility is then determined by the quality of the predictions it makes and the accuracy with which these accord with empirical observation. No natural scientist can ever really be excused, however, of believing or stating that such and such a theory is 'true' according to the most puritanical interpretation of that word, or 'represents reality' or other such sentiments. It is utility that is the distinguishing touchstone of the philosophy of natural science in its competition with other philosophies.

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The mathematician does not, by necessity, share this philosophical basis with the natural scientist, and yet the startling practical successes of natural science over the past 300 years have been achieved by generations of natural scientists working with the 'armoury' provided by mathematicians and still being vigorously expanded today. The natural scientists have 'subverted' mathematics for their own modest but determined aims: to construct models with the greatest and richest power to illuminate and draw inferences about the nature of postulated reality.

The mathematician's criterion of virtue is by contrast an aesthetic one and when working with the natural scientist this should be emphasized. We may perceive reality as beautiful and mathematics as beautiful but it would not be logically correct to infer that mathematics is reality, that the elegant solution is necessarily the best.

This should be commonplace among natural scientists, but the success of the scientific method over the past 300 years is such that these essential philosophical foundations are insufficiently well remembered. A particularly grotesque mutilation of scientific philosophy is performed by those who claim that particular scientific theories are true 'to all intents and purposes'. Newtonian mechanics explained the movements in the heavens to a degree sufficient for all practical purposes. However, quantitatively speaking the tiniest chink in the accordance of the theory's prediction with reality was sufficient to open the way for the einsteinian revolution. The discovery of such a chink had to await the considerable technological advances in optics and instrumentation of the nineteenth century before it could be perceived. The atomic bomb is not a negligible consequence of the tiniest imperfection in a 'nearly true' theory. Out of a tiny inconsistency between observed reality and an aesthetically complete theory arose not just enormous practical consequences, but a revolution in our understanding of the relation between the natural sciences and the reality that is perceived as their subject. In science, as in other areas of life, ladders must be climbed only so as to be kicked away. The father of natural science was Plato, whose notion of an absolute reality capable of limitless investigation through reason has inspired centuries of effort to elucidate this reality. Yet Heisenberg's, as yet unrefuted, theory is arguably consistent with the idea that such an absolute reality can never be observed. The idea that models are nearly true or are true for all practical purposes must be rejected as forcefully as the idea that they are true. If they look true we must look for the explanation. For them to be true in the strictest sense demands the impossibility of an observation out of accordance with them; a position unworthy of further discussion.

It is the empirical success of natural scientific philosophy more than any other body of ideas that has made the modern world much more than a mere continuation of the ancient. The industrial revolution is Great Britain would have been impossible without the spread of natural scientific philosophy in the 17th and 18th centuries. In a letter to Adam Smith commenting upon *The Wealth of Nations*, dated 10 September 1759 Edmund Burke wrote, 'A theory like yours, founded on the nature of man, which is always the same, will last, when those that are founded upon his opinions, which are always changing, will and must be forgotten.'

Natural science provides an independent method of arbitration between views, and this more than any other factor has enabled the replacement of sterile conflict and assertion with constructive argument and benign progress.

On the human timescale, however, we must not fail to appreciate that the philosophy of natural science is still new. Political, theological and economic organization and activity have thrived through the long ages of history under the sway of less modest philosophies and where political and theological ideologies or economic interest have collided with the timorous ambitions of the natural scientist, it is often the latter who, in the short term, has had to yield. These philosophies and ideologies influenced by the competitive successes of the natural sciences have sought the company of mathematics, perhaps in an attempt to bathe in the reflected light of reason. But mathematics has not prospered in such company as it has with the philosophy of natural science, which has provided it simultaneously with a raison d'être and a continual source of fresh stimuli. Mathematics does not need natural science, but it thrives on its company.

2. Scientific method and modelling market behaviour

Now let us apply these thoughts, slightly randomly, to developments in the mathematical modelling of market behaviour. I have already referred to Adam Smith who was among the first to formalize a model of the growth of economic and social systems founded on the philosophy of natural science. For those who know of market economics only by ill-repute it may perhaps come as a surprise to learn that Smith's mode of construction, as Burke's words testify, was deeply empirical, founded on detailed and analytical observation of the nature of human society. From this construction, the establishment of freely determined market prices in both agricultural and manufactured goods emerges as the optimum method for establishing values where the aim is to foster the maximum rate of that quantity, economic growth, which is seen as a natural consequence of a society in which people are free to interact economically and when the rule of law and the defence of property are guaranteed. This model, its laws of supply and demand and its concept of the division of labour have played the role slightly akin to that Newton's laws of motion played in physics in the development of the science of economics since. But although Newton's ideas were taken up to extraordinary effect by those studying a nature that does not talk back, in economics as in other social sciences, progress has been held back by the sheer practical difficulty of employing the experimental method. It has rarely been possible to experiment in a controlled fashion with society as a whole, and when something resembling experimental conditions are created, the 'adjudicator' of natural science is unlikely always to be respected by the larger part of the jury. Thus, today rather than being seen as the 'progenitor' of economic modelling, in the way that Stephenson is the 'progenitor' of the steam train or Brunel the 'progenitor' of civil engineering, Smith is seen as a 'political' figure and his excellent, and in my opinion, proven, contribution, scientifically undervalued. This is not to take sides in the modern political debate which is often concerned with multidimensional problems beyond the scope of Smith's work but is merely to observe that Smith's models did give rise to empirically testable, interesting and often counter-intuitive predictions about the world and which in the messy social laboratory of the past three centuries have received far more support than contradiction. That the experiments took a long time to perform, that they are always necessarily less conclusive than controlled experiments and that any analysis made of them will not necessarily be accepted by social scientists are reasons progress in the field of social and political science has been so much less impressive than in the physical or biological sciences. In the physical sciences the object of study does not have a mind of its own!

I mention Adam Smith because his models were constructed long enough ago for them to have been tested for robustness under a wide range of conditions. The evolution of social systems proceeds at a slower pace than that at which we are nowadays accustomed to living and thus it is much more realistic to judge the usefulness of models developed some time ago, without prior knowledge, than it is to assess their possible contemporary equivalents. Had Smith lived through the subsequent centuries there would have been many occasions on which he would have been able to profit handsomely by exploiting the difference between his theories' predictions and the common opinion and that is the ultimate judge of the utility of his model, of its value as a scientific theory.

I have stressed that a theory or model is scientifically valueless unless it is capable of making empirically falsifiable predictions. In economics and the study of markets there can be no more incorruptible measure of utility than profit accrued by arbitraging the predictions of a model against the common view (the market). The quest to 'beat the market' is thus more than a venal desire for money or an egotistical desire to win, it is also a means of obtaining the most ruthlessly honest evaluation of the scientific utility of a model or method.

Speculari, the Latin root of the verb to speculate, has the literal meaning 'to observe'. And a study of speculation will show that most successful speculators can be well described as 'observers'. To be successful, this observation must of necessity be detached and unemotive and thus, where great social and moral issues are at stake, it is perhaps not surprising that this viewpoint should arouse some distrust and hostility among the general population (particularly when the speculator profits at a time of general discontent). Yet this detached observation is clearly in the spirit of the natural scientist and the act of speculating for money is in the spirit of the empirical scientist's restless yearning to add to empirical knowledge and put theories to the test. Thus, making money from mathematical models is in one sense less about the corruption of intellectual endeavour than about the appropriate statistical test of the utility of such models for the development of scientific theory.

3. The utility of the efficient market theory

It is in this context that I wish to consider the scientific utility of the 'efficient market theory'. The various versions of the theory begin essentially by asserting that it is impossible to make money by applying mathematical modelling to the science of speculation. What, then, is its utility as a scientific theory? On the one hand its predictions of market price are of the null variety – that no better estimate of tomorrow's price than today's can be discerned – and not very interesting. On the other hand, a concrete prediction that future returns will be drawn from a known distribution whose parameters can be estimated appears falsified; the empirical evidence points to the return process in all markets being ultimately non-parametric and certainly non-stationary. Its great strength is that it is consistent with one of the most profoundly useful insights about market behaviour: it is very difficult to make money consistently. Such consistency, however, it not a unique feature of this model over a universe of alternatives.

What of the practical evidence? Because every major bank and securities house now has its option software and its rocket scientists surely they must be making money from the models thus indirectly confirming their utility. But how is this money made? First through arbitrage – using the model to assess the relative value of various forms of derivatives of the same asset or assets – a test that is relatively insensitive to the crucial distributional assumptions underlying the theory; second, through what we may (not necessarily derogatorily) call merchandising: banks and brokers selling at marked up prices derivative instruments that can only be created because of the existence of the theory. These profits do not ultimately refute the theories' scientific utility. If this sounds contrived consider the case of portfolio insurance. Some made personal fortunes from selling advice based on the theory (in good conscience) but to compensate, after the market crash of 1987, their pension fund clients incurred losses greater than they otherwise would have done. Thus the widespread use, found for the theory, is not strong evidence of its scientific utility but more for its marketability. None of this is to denigrate the contribution of the theory towards improved practice in and greater understanding of investment but it is to point out sharply its limited ambition and limited utility as a scientific theory and to undermine the perception of confirmation its widespread usage suggests. It is no surprise of course that speculators should be in conflict with the theory because it explicitly denies the possibility of their existence.

4. Conclusion

Having aired my doubts I now have some positive comments on how to make money from mathematical models or perhaps more properly how I have observed money being made. I believe there are three distinct paths that can be followed.

1. Be a purveyor of derivative instruments or shareholder in such an activity. This has all the intellectual purity of selling vegetables!

2. Be a rocket scientist arbitrageur. The efficient market theory is sufficiently robust with respect to relative values and some very challenging mathematics has been required to unlock new arbitrage potential in the globalizing financial markets. This can be challenging and satisfying but its assumptions may be unsound.

3. Speculate, which I believe to be the intellectual front line. One may study and observe the world so as to seek phenomena amenable to classification and to form ideas as to the metaphors and analogies that are components of a model that can make interesting and falsifiable predictions.

The disincentive to a mathematician of pursuing course 3 is that observation and classification are not the mathematician's job! But any mathematician motivated by the philosophy of science will not find a shortage of opportunity in this course. Efficient market theory has at least partly driven the charlatan from the investment stage. Perhaps this has created the opportunity for the scientist to take to that stage and to push further back the frontier of ignorance for the betterment of humankind.